Exercise IX

1. Use the Intermediate Value Theorem to prove that:
   (i) there is a real number c such that $c^2 = 5$;
   (ii) $x^3 - 3x^2 + 10x - 7$ has a zero in the interval [0,1];
   (iii) if $f(x) = x^3 - x^2 + x$, then there is $c \in \mathbb{R}$ such that $f(c) = 10$.

2. Sketch the graph of a (non-constant) function which is continuous over [-2,4] and differentiable over (-2,4) and

   (i) has its maximum and minimum value in (-2,4);

   (ii) has its maximum value in (-2,4) and minimum value at an end point of the interval [-2,4].

   (iii) Has its minimum value in (-2,4) and maximum value at an end point of the interval [-2,4].

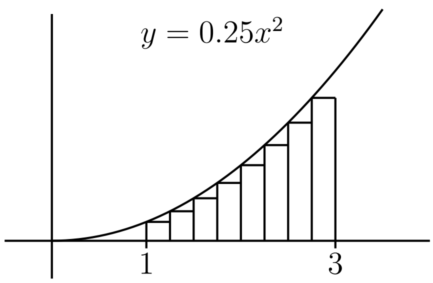
   (iv) Has its maximum value at an end-point of -[2,4] and a minimum value at an end-point of [-2,4].

3. Sketch the graph of a function that does not have a maximum or a minimum value over [-2,4].

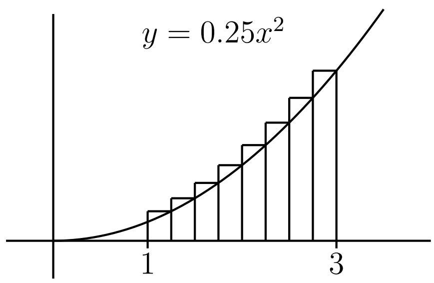
4. Sketch the graph of a function which has a maximum value at some point $c \in (-2, 4)$ but $f'(c) \neq 0$.

5. Sketch the graph of a function which has a minimum value at some point $c \in (-2, 4)$ but $f'(c) \neq 0$

6. Determine the total area of the rectangles illustrated in (i) and (ii) respectively:



(i)



(ii)